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RELIABILITY TEST PLAN FOR THE NEW WEIBULL-PARETO DISTRIBUTION

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Abstract: **Sampling plans in which items that are put to test, to collect the life of the items in order to decide upon accepting or rejecting a submitted lots are called reliability test plans. In this paper, a reliability test plan is used for acceptance or rejection of a lot of products submitted for inspection in which the lifetime of the items follow New Weibull-Pareto Distribution. Here the proposed sampling plan can save the test time in practical situations. Minimum sample size required to accept or reject a submitted lot for a given acceptance number with producer's risk is found. The test plan to determine the termination time of the experiment for a given sample size, producer's risk and termination number is also constructed. The comparison of the reliability test plan over similar plans is established with respect to time of the experiment. Results are illustrated by an example.**

Keywords: **New Weibull-Pareto Distribution, Reliability Test Plan, Minimum Sample Size, Producer's risk, Experimental time.**

1. INTRODUCTION

Acceptance Sampling Plans in Statistical Quality Control are concerned with accepting or rejecting a submitted lot of a large size of products on the basis of the quality of the products inspected in a sample taken from the lot. Acceptance sampling, if it is applied to a series of lots, prescribes a procedure that will give a specified probability of accepting lots of given quality. It is the process of inspecting a sample of product/material drawn from the lot to accept or reject the lot as either conforming to or not conforming to quality specifications i.e., inspection based on qualitative measurements. Inspection for acceptance purpose is used at various states in manufacturing process. Sampling inspection can be defined as a technique to determine the acceptance or rejection of a lot or population by some defective parts found in a random sample drawn from the lot. If the number of defective items does not exceed a predefined level, the lot is accepted. Otherwise, it is rejected.

In acceptance sampling inspection a defective article is defined as one that fails to conform to specifications in one or more quality characteristics. A common procedure in acceptance sampling is to consider each submitted lot of product separately and to base the decision on acceptance or rejection of the lot on the evidence of one or more samples chosen at random from the lot. If the quality of the product is the lifetime of the product, then a sample of such products, considered to be representive of the observed lifetimes of the products are put for testing. If a decision to accept or reject the lot, subjected to the risks associated with the two types of errors (rejecting a good lot / accepting a bad lot), is possible then such a procedure is termed as 'acceptance sampling based on life tests' or 'reliability test plans'.

Let us assume that the lifetime of a product follows New Weibull-Pareto distribution, given by Suleman Nasiru and Albert Luguterah (2015), whose probability density function and cumulative distribution function are given respectively by

$$f(t;\beta,\sigma,\delta)=\frac{\beta\delta}{\sigma}\left(t/\sigma\right)^{\beta-1}e^{-\delta\left(t/\sigma\right)^{\beta}};0<t<\infty,\beta>0,\sigma>0,\delta>0-----(1)$$

$$F(t;\beta,\sigma,\delta)=1-e^{-\delta\left(t/\sigma\right)^{\beta}};0<t<\infty,\beta>0,\sigma>0,\delta>0-----(2)$$

where σ is the scale parameter and its two shape parameters β and δ are both positive. Acceptance sampling plans based on truncated life tests for exponential distribution was first discussed by Epstein [2]. The results were extended for the Weibull distribution by Goode and Kao [3], Balakrishnan et al. [1] provide the time truncated acceptance plans for Generalized Birnbaum-Saunders Distribution, Kantam, Rosaiah and Srinivasa Rao [4] developed acceptance sampling based on life tests: log-logistic model. Srinivasa Rao, Ghitany and Kantam [5] developed Reliability Test Plans for Marshall-Olkin Extended Exponential Distribution. Srinivasa Rao [6] developed an Economic Reliability Test Plan Based on Truncated Life Tests for Marshall-Olkin Extended Weibull Distribution. The Reliability Test plan is given in section 2. The comparative study is presented in section 3. The conclusion is given in section 4.

2. RELIABILITY TEST PLANS

To compare the performance of various acceptance sampling plans, their performance over a range of possible quality levels is studies. In statistical quality control, acceptance sampling plan is concerned with the inspection of a sample of products taken from a lot and the decision whether to accept or not to accept the lot based on the quality of the product. In a life testing experiment, the procedure is to terminate the test by a predetermined time t and note the number of failures. If the number of failures at the end of time t does not exceed a given number c, called acceptance number then we accept the lot with a given probability of at least p. But if the number of failures exceeds c before time t then the test is terminated, and the lot is rejected. For such truncated life test and the associated decision rule, we are interested in obtaining the smallest sample size to arrive at a decision.

In the sequel, we assume that the distribution parameters β and δ are known, while σ is unknown. In such a case, the average lifetime of the product depend only on σ, and can be observed that the average lifetime is monotonically increasing in σ. Let σ_0 represent the required minimum average lifetime, then, for given values of β and δ.

The consumer's risk, i.e., the probability of accepting a bad lot should not exceed 1-P^*, where P^* is a lower bound for the probability that a lot of true value of σ below σ_0 is rejected by the sampling plan. For a fixed P^*, sampling plan is characterized by (n, c, t/σ_0).

By sufficiently large lots we can apply binomial distribution to find acceptance probability. The problem is to determine the smallest positive integer n, for given values of P^*, σ_0 and c, such that

$$L(p_0)=\sum_{i=0}^{c}\binom{n}{i}p_0^{\,i}(1-p_0)^{n-i}\le 1-p^*-----(3)$$

Where p_0 = F(t; β, δ, σ_0), obtained from equation (2), indicates the failure probability before time t depends only on the ratio t/σ_0. The function L(p) is the operating characteristic function of the sampling plan, i.e., the acceptance probability of the lot as function of the failure probability p(σ)=F(t;β,δ,σ) is decreasing function in σ which implies that the operating characteristic function is increasing in σ. The minimum values of n satisfying the inequality (3) are obtained and displayed in Table 1 for P^*= 0.90, 0.95, 0.99 and t/σ_0= 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 for β = δ = 2.

Table 1: Minimum sample size required to accept / reject a submitted lot for a given acceptance number with producer's risk P^* using binomial approximation

p	c	t/σ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.90	0	3	2	1	1	1	1	1	1
	1	6	3	2	2	2	2	2	2
	2	8	5	4	3	3	3	3	3
	3	11	6	5	4	4	4	4	4
	4	13	7	6	5	5	5	5	5
	5	15	9	7	6	6	6	6	6

	6	17	10	8	7	7	7	7	7
	7	19	11	9	8	8	8	8	8
	8	21	13	10	9	9	9	9	9
	9	23	14	11	10	10	10	10	10
	10	25	15	12	11	11	11	11	11
0.95	0	4	2	1	1	1	1	1	1
	1	7	4	3	2	2	2	2	2
	2	9	5	4	3	3	3	3	3
	3	12	7	5	4	4	4	4	4
	4	14	8	6	5	5	5	5	5
	5	16	9	7	6	6	6	6	6
	6	19	11	8	7	7	7	7	7
	7	21	12	9	9	8	8	8	8
	8	23	13	11	10	9	9	9	9
	9	25	15	12	11	10	10	10	10
	10	27	16	13	12	11	11	11	11
0.99	0	6	3	2	1	1	1	1	1
	1	9	5	3	2	2	2	2	2
	2	12	6	4	4	3	3	3	3
	3	14	8	6	5	4	4	4	4
	4	17	9	7	6	5	5	5	5
	5	19	11	8	7	6	6	6	6
	6	21	12	9	8	7	7	7	7
	7	24	13	10	9	8	8	8	8
	8	27	15	11	10	9	9	9	9
	9	28	16	12	11	10	10	10	10
	10	31	18	14	12	11	11	11	11

Alternatively, we considered another approach for a reliability test plan. We summarize this approach. Let n indicates the number of sampled items to be determined and r stands for a natural number, such that if r failures out of n samples are occurred before the terminated time t the lot would be rejected. In this aspect, r is called as termination number. The sample size is depending upon the cost consideration and the expected time to reach a decision. If the sample size is large it may reduce the expected waiting time but increase the cost of consideration. Let us take sample size as a multiple of the termination number to balance between these two aspects. As we have come to know that the probability of r failures out of n tested items is given as ${}_nC_r p^r(1-p)^{n-r}$, where p=F(t;β,δ,σ) as before.

Thus, the probability of accepting the lot is

$$L(p)=\sum_{i=0}^{r-1}\binom{n}{i}p^i(1-p)^{n-i} \text{------} (5)$$

If α is producer's risk then equation (6) can be written as:

$$\sum_{i=0}^{r-1}\binom{n}{i}p^i(1-p)^{n-i}=1-\alpha \text{------} (6)$$

Given the values of n=r.k, equation (6) can be solved for p using cumulative probabilities of binomial distribution. Then the values of p can be used in equation (6) for α = 0.10, 0.05, 0.01 to find the values of t /σ. These values for different values of r and n are given in Table 2.

Table 2: Reliability Test Plan for New Weibull-Pareto Distribution for $\beta = \delta = 2$.

$\alpha = 0.10$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.16229	0.13251	0.11476	0.10264	0.09370	0.08675	0.08114	0.07650	0.07258
2	0.27731	0.22041	0.18854	0.16744	0.15215	0.14040	0.13102	0.12329	0.11679
3	0.33487	0.26333	0.22421	0.19861	0.18017	0.16607	0.15484	0.14562	0.13788

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4	0.37012	0.28935	0.24577	0.21742	0.19707	0.18154	0.16920	0.15907	0.15057
5	0.39436	0.30716	0.26051	0.23026	0.20860	0.19210	0.17899	0.16824	0.15923
6	0.41226	0.32028	0.27136	0.23972	0.21709	0.19987	0.18620	0.17500	0.16560
7	0.42614	0.33045	0.27976	0.24705	0.22367	0.20589	0.19178	0.15023	0.17054
8	0.43730	0.33861	0.28651	0.25293	0.22895	0.21073	0.19627	0.18443	0.17450
9	0.44652	0.34535	0.29208	0.25778	0.23331	0.21472	0.19997	0.18790	0.17778
10	0.45428	0.35103	0.29677	0.26188	0.23699	0.21808	0.20309	0.19082	0.18054
$\alpha = 0.05$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.11324	0.09246	0.08007	0.07162	0.06538	0.06053	0.05662	0.05338	0.05064
2	0.22662	0.18016	0.15411	0.13686	0.12436	0.11477	0.10710	0.10078	0.09547
3	0.28831	0.22678	0.19311	0.17106	0.15519	0.14305	0.13337	0.12543	0.11876
4	0.32735	0.25601	0.21747	0.19239	0.17438	0.16065	0.14972	0.14076	0.13324
5	0.35468	0.27637	0.23442	0.20721	0.18772	0.17288	0.16108	0.15141	0.14329
6	0.37512	0.29155	0.24704	0.21825	0.19766	0.18198	0.16953	0.15933	0.15078
7	0.39112	0.30342	0.25691	0.22688	0.20542	0.18909	0.17614	0.16552	0.15663
8	0.40408	0.31302	0.26489	0.23385	0.21169	0.19484	0.18147	0.17053	0.16135
9	0.41484	0.32098	0.27151	0.23964	0.21689	0.19961	0.18590	0.17468	0.16527
10	0.42935	0.32773	0.27711	0.24453	0.22130	0.20365	0.18965	0.17819	0.16859
$\alpha = 0.01$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.05013	0.04093	0.03545	0.03171	0.02894	0.02680	0.02506	0.02363	0.02242
2	0.14648	0.11647	0.09964	0.08849	0.08041	0.07421	0.06925	0.06577	0.06173
3	0.21041	0.16558	0.14101	0.12492	0.11333	0.10446	0.09741	0.09160	0.08673
4	0.25390	0.19867	0.16879	0.14933	0.13536	0.12470	0.11622	0.10927	0.10343
5	0.28553	0.22262	0.18886	0.16695	0.15125	0.13929	0.12979	0.12200	0.11567
6	0.30975	0.24090	0.20416	0.18038	0.16336	0.15041	0.14012	0.13169	0.12463
7	0.32906	0.25544	0.21634	0.19106	0.17299	0.15925	0.14834	0.13940	0.13191
8	0.34490	0.26736	0.22628	0.19979	0.18086	0.16647	0.15505	0.14570	0.13786
9	0.35818	0.27733	0.23463	0.20710	0.18745	0.17252	0.16067	0.15097	0.14285
10	0.36952	0.28584	0.24174	0.21334	0.19308	0.17768	0.16547	0.15547	0.14710

As an example of this approach, let us think that we have to derive a life test sampling plan with an acceptance probability of 0.95 for lots with an acceptable mean life of 1000 hours and 10, 5 as sample size, termination number r respectively. From table 2, the entry against r=5 under column 2r is 0.35468. This implies that the termination time t = 354.68 hours. In this test plan, we select 10 items from the submitted lot and put to test. We reject the lot, when the 5th failure is occurred before 354.68 hours, otherwise we accept the lot. In either case terminating the experiment as soon as the 5th failure occurs or the termination time 354.68 hours is reached, or whichever is earlier.

Table 3: Comparison of Proportion of termination time

r \ n	2r	3r	4r	5r	6r
α = 0.05					
1	0.11324 0.942		0.08007 0.628		
2	0.22662 0.942				
3	0.28831 0.628				
4		0.25601 0.628			
α = 0.01					

1	0.05013 1.257	0.04093 0.942			0.02894 0.628
3	0.21041 0.942		0.14101 0.628		
4	0.25390 0.942				
7		0.25544 0.628			
8		0.26736 0.628			
9		0.27733 0.628			

3. COMPARATIVE STUDY

The upper entry in each occupied cell of Table 3 corresponds to the proportion of termination time of the reliability test plan given in table 2. The lower entry corresponds to the similar quantity of the acceptance sampling plan given in table 1, respectively. These entries reveal that the terminating time of the reliability test plan given in table 2 is uniformly smaller than the corresponding time of the sampling plan given in table 1 and also it would be beneficial in terms of test time and cost.

4. CONCLUSION

In this paper a reliability test plan is developed when the lifetimes of the items follow the New Weibull-Pareto distribution. Minimum sample size required to accept or reject a submitted lot for a given acceptance number with producer's risk were obtained. Values of termination time for the given values of sample size were provided. On comparing both the approaches of sampling plans presented in this paper, the reliability test plan of second approach would result in many savings in the experimental time and cost.

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